

Second law, exergy, and consequences on low-temperature heat-recovery

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Energy Management for Large-Scale Research Infrastructures



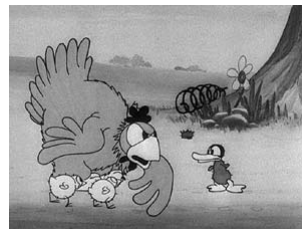
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Tribute to H.C. Andersen

(April 1805, Odense - August 1875, Copenhagen)

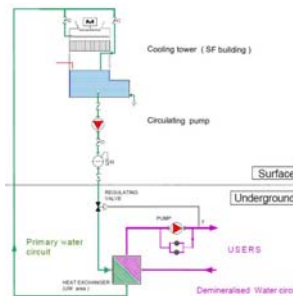
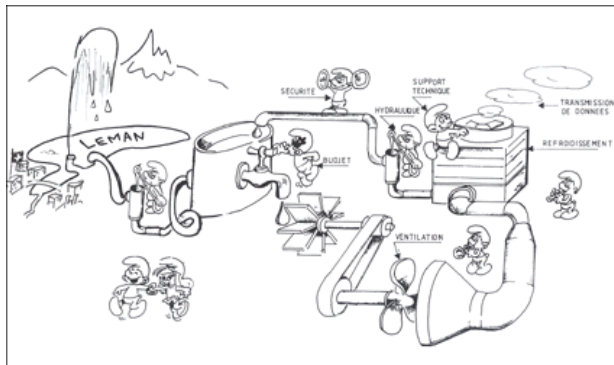
and especially to the tale

The Ugly Duckling



Can one produce energy out of a huge amount of heat delivered at 40-50°C?

- 23 cooling towers in running condition exist at CERN. These are situated all over the CERN territory with cooling power ranging from 1.25 MW to 70 MW.



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Can one produce energy out of a huge amount of heat delivered at 40-50°C?

- *Immediate answer: NO*
 - ANY GOOD REASON?
 - *Yes, the second law!*
 -
- So, come and tell us.



• Summary

- The first law and its consequences on energy conversion systems
- The second law and its consequences on energy conversion systems
- Exergy (entropy and irreversibility)
- Consequences on energy recovery at low temperature

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Framework

- Macroscopic systems ...
 - globally out of equilibrium,
 - at local thermodynamic equilibrium at microscopic level.
 - *Mechanics of continuous media*
- Systems in periodic regime
 - Changes in internal quantities have zero integrals
- Exchanges with *constant*-characteristics sources:
 - Heat <-> with thermostats (constant *Temperature*)
 - Mass <-> infinite reservoirs (constant *Pressure, composition* ...)

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First law: total energy is conserved

- The change in total energy equals the inputs of work plus heat. $dE = \delta W + \delta Q$
- Work=action of forces
Heat=gradient of heat flux $\frac{\partial(\rho e)}{\partial t} + \nabla \cdot (\rho e \mathbf{v}) = -\nabla \cdot (\mathbf{P} \cdot \mathbf{v} + \mathbf{q}) + \rho \mathbf{F} \cdot \mathbf{v}$
- Total energy = internal energy + kinetic energy: $E = U + E_k$
 $e = u + \mathbf{v} \cdot \mathbf{v} / 2$

$$\rho(\dot{u} + \mathbf{v} \cdot \dot{\mathbf{v}}) + (u + \mathbf{v} \cdot \mathbf{v} / 2)(\dot{\rho} + \rho \nabla \cdot \mathbf{v}) = -\nabla \cdot \mathbf{q} - (\nabla \cdot \mathbf{P}) \cdot \mathbf{v} - \mathbf{P}^T : \nabla \mathbf{v} + \rho \mathbf{F} \cdot \mathbf{v}$$

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0$$

$$\rho \dot{\mathbf{v}} + \nabla \cdot \mathbf{P} - \rho \mathbf{F} = 0$$

$$\rho \dot{u} = -\nabla \cdot \mathbf{q} - \mathbf{P}^T : \nabla \mathbf{v}$$

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Consequences on energy conversion systems

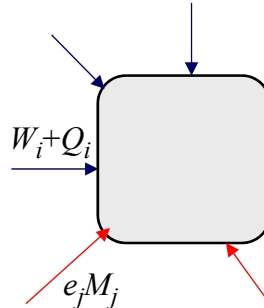
- Consider a machine for energy conversion operated periodically and exchanging energy fluxes W_i and Q_i with different sources:

$$\sum_i (W_i + Q_i) = 0$$

- If also exchanging mass fluxes, M_j with average energy e_j :

$$\sum_j (M_j) = 0$$

$$\sum_i (W_i + Q_i) + \sum_j (e_j M_j) = 0$$



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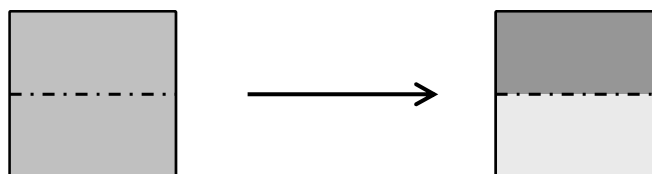


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Now imagine ...

- ... 1+1 kg of steel initially at uniform temperature T_0 ...

... and with final state = 1 kg at $T_0 + DT$ and 1 kg at $T_0 - DT$.



- Everyone knows that such a transfer *never occurs spontaneously*, although it would perfectly conserve energy. Then, Why?
 - Because of the *second law*.

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The second law (1)

- Introduction: There exists a function of state s from which (macroscopic) temperature T can be defined such that:

$$Tds = du - \frac{p}{\rho^2} d\rho - \sum_k \mu_k dc_k \quad (= du - \delta w = \delta q)$$

- The transport equation for **entropy** is:

$$\frac{\partial}{\partial t}(\rho s) + \nabla \cdot (\rho s \mathbf{v}) = -\nabla \cdot \left[\frac{1}{T} \left(\mathbf{q} - \sum_k \mu_k \mathbf{J}_k^d \right) \right] + \delta s^+$$

Conductive heat flux density
mass flux density
↑

- **Non-zero source term: entropy is not conserved.**

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The second law (2)

- **Statement of the second law:**
the source term σ^s cannot be negative

$$\delta s^+ = \mathbf{q} \cdot \nabla \left(\frac{1}{T} \right) - \frac{1}{T} p^v \nabla \cdot \mathbf{v} - \frac{1}{T} \mathbf{P}^v : \mathbf{V} - \sum_k \mathbf{J}_k^d \cdot \nabla \left(\frac{\mu_k}{T} \right) + \frac{\rho}{T} \sum_l \mathcal{A}_l \dot{\xi}_l + \frac{1}{T} \boldsymbol{\varepsilon} : \mathbf{i}$$

- **Five irreversible phenomena:**
Conduction - friction (bulk and shear)
mass diffusion - chemical reactions - Joule effect

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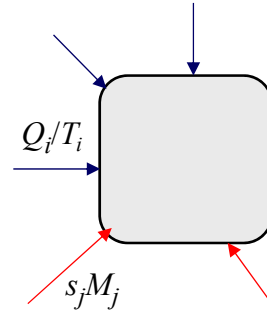
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Balances in energy conversion systems

$$\sum_j (M_j) = 0 \quad \sum_i (W_i + Q_i) + \sum_j (e_j M_j) = 0$$

- Heat sources at constant temperatures T_i exchanging **entropy fluxes** Q_i/T_i
- mass fluxes M_j with average entropy s_j :



$$\sum_i (Q_i / T_i) + \sum_j (s_j M_j) + \Delta S^+ = 0$$

$$\Delta S^+ \geq 0$$

- $\Delta S^+ = 0 \iff$ Reversibility

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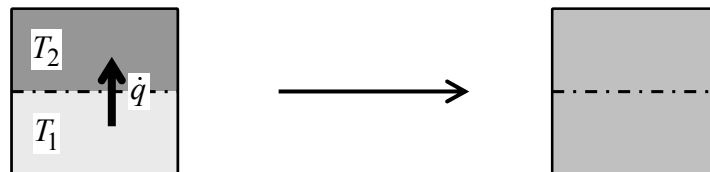
Heat transfer from T_1 to T_2

- The same two kilograms of steel (*e.g.*), one at T_1 , one at T_2 , with $T_1 < T_2$, and \dot{q} the heat flux from 1 to 2 ...

By virtue of the second law $\delta s^+ = \dot{q}(T_2^{-1} - T_1^{-1}) \geq 0$

one has: $\dot{q} \leq 0$

Heat is transferred from 'hot' (T_2) to 'cold' (T_1) leading to uniformity.



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More general expression of the second-law

- None of the six terms can be negative:

$$\delta S^+ = \mathbf{q} \cdot \nabla \left(\frac{1}{T} \right) - \frac{1}{T} p^v \nabla \cdot \mathbf{v} - \frac{1}{T} \mathbf{P}^v : \mathbf{V} - \sum_k \mathbf{J}_k^d \cdot \nabla \left(\frac{\mu_k}{T} \right) + \frac{\rho}{T} \sum_l \mathcal{A}_l \dot{\xi}_l + \frac{1}{T} \boldsymbol{\varepsilon} \cdot \mathbf{i}$$

- Coefficients such as conductivity, viscosity (bulk- and shear-), diffusion coefficient, chemical affinity and resistivity *cannot be negative*.

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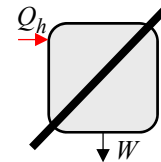


Consequences on energy conversion systems

- Can a heat engine produce mechanical work out of one heat source only?

$$\left. \begin{aligned} Q_h = -W \geq 0 \\ \Delta S^+ = -Q_h / T_h \geq 0 \end{aligned} \right\} \Rightarrow Q_h = 0$$

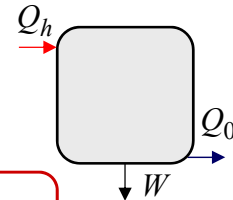
- No!



- There must be (at least) two heat sources, one of them is the environment (subscript 0):

$$\left. \begin{aligned} Q_h + Q_0 + W = 0 \\ \frac{Q_h}{T_h} + \frac{Q_0}{T_0} + \Delta S^+ = 0 \\ \Delta S^+ \geq 0 \\ W \leq 0 \end{aligned} \right\} \Rightarrow \begin{cases} (T_h \geq T_0) \\ Q_h \geq 0 \\ Q_0 \leq 0 \end{cases}$$

$$\frac{-W}{Q_h} = 1 - \frac{T_0}{T_h} - \frac{\Delta S^+ T_0}{Q_h} \leq 1 - \frac{T_0}{T_h} < 1$$



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Introduction of exergy

- Total energy is conserved, but not any energy can be *entirely* converted into work.
- The conversion efficiency is bounded by an **upper limit** called **the Carnot efficiency** $= 1 - T_0/T_h$ (the Carnot cycle is reversible, *i.e.* with $\Pi_S=0$).
- This upper limit is obtained by combining the balances of total energy E and of entropy S , according to: $\underline{E_{\text{tot}} - T_0 \cdot S}$.
 T_0 = temperature of the heat source where heat can be released (or from which heat can be extracted) for free,
= temperature of the environment (outdoor air).
- The quantity $B = E_{\text{tot}} - T_0 \cdot S$ is called **EXERGY**.

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Definition of exergy

- Exergy =
the **maximal amount of mechanical work**
that can be **produced** when using,
either a *flux of energy*⁽¹⁾, or a *given mass*⁽²⁾,
by **reversible processes** operated in a **given environment**
(temperature T_0 , pressure, composition, etc.).
- Notation: B (or b ...) – Unit: J (or $\text{J} \cdot \text{kg}^{-1}$...)

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Definition of exergy (2)

- Exergy = maximal amount of mechanical work produced from, either a flux of energy⁽¹⁾, or a given mass⁽²⁾, by reversible processes in a given environment (temperature T_0 , pressure, composition, etc.).
- (1) considering a flux of energy E characterized by a flux of entropy S , then the flux of exergy B is:

$$\dot{B} = \dot{E} - T_0 \dot{S} = \dot{W} + \dot{Q} - T_0 \dot{Q} / T$$

$$\dot{B} = \dot{W} + (1 - T_0 / T) \dot{Q}$$
- (2) considering mass, exergy = state function, with changes:

$$db = de - T_0 ds = de_k + du - T_0 \left(T^{-1} du + T^{-1} p \rho^{-2} d\rho + T^{-1} \sum_k \mu_k dc_k \right)$$

$$db = de_k + (1 - T_0 / T) du + (T_0 / T) \left(p \rho^{-2} d\rho + \sum_k \mu_k dc_k \right)$$
 - Note: any term related to heat (Q , du) is multiplied by $(1 - T_0/T)$, called the *Carnot factor*.

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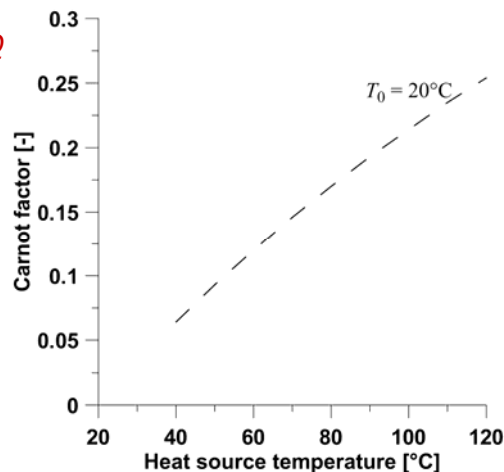
The present concern: to produce work out of heat

$$(1 - T_0/T) = \text{Carnot factor} = B/Q$$

Efficiency of a *Carnot* cycle operated between sources at temperatures T and T_0 , (T_0 : environment, T : point under consideration).

B/Q = Conversion efficiency of reversible engine cycles with a heat source at T = upper limit.

Here with environment at 20°C



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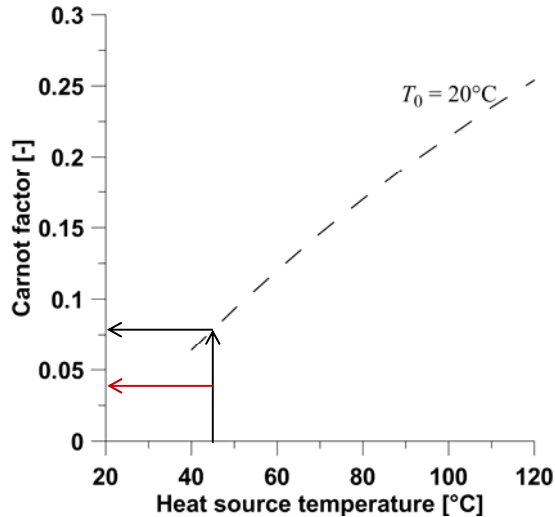
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Heat flux Q released at 45°C (1)

- Carnot factor $B/Q = 0.08$
- The exergy content of heat delivered at 45°C in environment at 20°C is only 8%:
only less than 8% can be converted into work.
- Considering now a **real process** heat to electricity: the likely efficiency $W_e/Q_h \approx 0.04$ (4% only of Q_h is converted into electricity).



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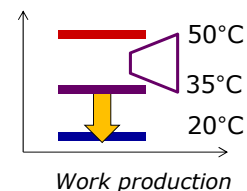
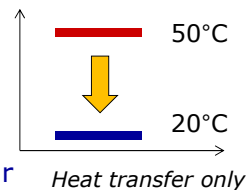
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Heat flux Q released at 45°C (2)

- **96% of Q_h must still be rejected to the environment** (negligible saving) ...
- ... but with a temperature difference between the heat-exchanger and the ambient air now reduced by a factor 2, if not more;
- **Consequences**
 - 1: much larger heat-exchange surface with ambient air : $S = Q/(h \cdot \Delta T)$,
 - 2: larger air flow-rate for heat rejection,
 - 3: *larger electricity consumption in fans.*
- The net benefit may be very limited (if not considering investment).



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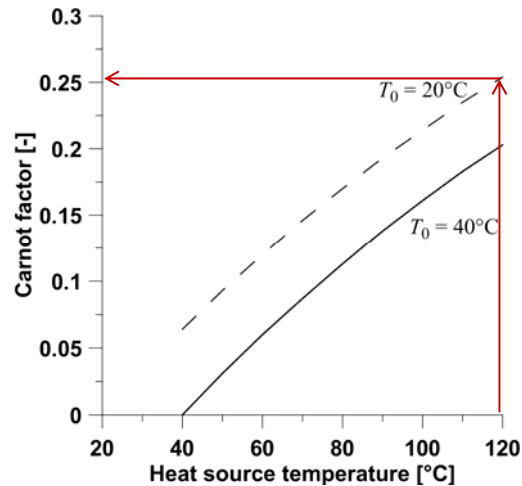
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Any solution?

- Usually, low-grade heat-recovery is achieved with heat delivered at 90-100°C, and preferably more (120°C).
- At 120°C, the exergy content is 0.25, then one can hope to obtain an effective conversion efficiency of 10% (still low but non-negligible).



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About process efficiency

- Conversion efficiency:

$$\frac{-W}{Q_h} = \left(1 - \frac{T_0}{T_h}\right) \left(1 - \frac{\Delta S^+ T_0}{Q_h (1 - T_0/T_h)}\right) = \left(1 - \frac{T_0}{T_h}\right) \left(1 - \frac{\Delta B^-}{B_h}\right)$$

- More generally:

$$Eff = Eff_{rev} \cdot \left(1 - \frac{\Delta B^-}{B_{in}}\right)$$

$$\delta b^- = T_0 \mathbf{q} \cdot \nabla \left(\frac{1}{T}\right) - \frac{T_0}{T} p^v \nabla \cdot \mathbf{v} - \frac{T_0}{T} \mathbf{p}^v : \overset{0}{\nabla} \overset{0}{\mathbf{v}} - T_0 \sum_k \mathbf{J}_k^d \cdot \nabla \left(\frac{\mu_k}{T}\right) + \frac{T_0 \rho}{T} \sum_l \mathcal{A}_l \dot{\xi}_l + \frac{T_0}{T} \boldsymbol{\varepsilon} : \mathbf{i}$$

with $(\delta b^- \geq 0)$

$$B_{in} = B_{out} - \Delta B^-$$

- Using exergy losses (or entropy productions), one can compare on a same basis (change in process efficiency) the effects of reducing this, or that, irreversibility: **the process can be optimized** (e.g. at given power).

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Conclusion

- Because of the temperature level, the exergy content of heat rejected by cooling towers is so low that (re-)conversion into electricity is not worth.
 - **Two possible ways:**
 - Recover heat at high temperature levels, *i.e.* up-hill the cooling tower loops (but still with limited conversion potential),
 - **Save energy** (probably larger potential) :
 - *Increase insulation of the zones which are at high or low temperature*
 - *Improve energetic efficiency of electricity consuming devices, auxiliaries, components, etc.*

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Thank you

for your attention,

and your questions.



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